BEHAVIOR OF THE COEFFICIENT OF FRICTION DURING THE FLOW OF COMPRESSIBLE GAS WITH VERY HIGH NEGATIVE PRESSURE GRADIENTS

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Zhurnal Prikladnoi Mekhaniki i Technicheskoi Fiziki, Vol. 7, No. 6, pp. 93–96, 1966

It is shown that for sufficiently large negative pressure gradients the coefficient of friction should increase independently of whether the flow is "laminarized" or not.

NOTATION

ζ is the coefficient of friction, M is the Mach number, λ is the velocity coefficient, τ is the frictional stress, p is the pressure, R is the Reynolds number, S is the entropy, k is the isoentropy exponent, T is the thermodynamic temperature, θ is the stagnation temperature, ω is the velocity-profile shape factor, U_* is the limiting velocity, w is the velocity of one-dimensional flow, u is the longitudinal velocity component, v is the lateral velocity component, x is the distance in the longitudinal direction, y is the distance in the lateral direction, D is the tube diameter, r is the tube radius, μ is the dynamic viscosity, ν is the kinematic viscosity, and ρ is the density.

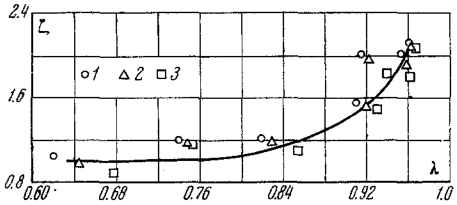

Fig. 1. Coefficient of friction ($\zeta = \xi$) as a function of the velocity coefficient according to direct measurements of the frictional stress at the wall.

The experiments described in [1] were the first to establish reliably the dependence of the "one-dimensional" coefficient of friction $\zeta_0 = 8\tau_0/\rho w^2$ on the Mach number M for a gas flowing in tube with subsonic velocity. In this relation the velocity w was determined from the one-dimensional flow equation, and the wall friction τ_0 was found from the one-dimensional angular momentum equation. The rapid decrease of the coefficient of friction before the onset of the critical state led to the laminarization hypothesis [2]. The laminarization phenomenon has, in fact, been detected in some experiments [3, 4]. Moreover, the theory of the boundary layer [5, 6] suggests that, for large negative pressure gradients, the coefficient of friction should increase both in turbulent and laminar flows.

This leads to the formulation of the following problem: it is required to find the actual (and not merely "one-dimensional") coefficient of friction for a gas flowing in a tube.

We have carried out direct experimental determinations of τ_0, and the results of these measurements were, of course, unaffected by the laminarization effect. We have used a vertical cylindrical tube 2735 mm long and 14.05 mm in diameter. The working section was 300 mm long and had the same diameter. A movable element was inserted through a slot in the working section, and measurements were made of its displacement under the action of frictional forces and pressure differences. To obtain the values of τ_0 for λ between 0.6 and 1.0, additional loads were placed on the movable element.

The results of measurements are shown in Fig. 1 in the form of $\xi = \zeta/\zeta^\circ$ as a function of λ, where ζ° corresponds to the incompressible fluid. In Fig. 1 the open circles, triangles, and squares represent different experiments. The experiments described in [1] have shown that ζ_0 should appreciably decrease for $\lambda > 0.80$. It is clear from Fig. 1 that ξ has a definite tendency to increase for $\lambda > 0.80$. Thus, for $\lambda = 0.95$ the ratio ξ exceeds 2, while ξ_0 is approximately equal to 0.55 in accordance with the data reported by MO TsKTI. The discrepancy by a factor of more than 3.5 can hardly be explained by some initial factors. For sufficiently large λ the velocity profile appears to undergo a substantial deformation. The values of λ beyond which an appreciable change in the velocity begins can readily be found from simple considerations.

Let us now find the rate of change of the entropy along the tube axis:

$$\frac{dS_1}{dx} = -\frac{1}{p}\frac{dp}{dx} + \frac{k}{k-1}\frac{1}{T_1}\frac{dT_1}{dx}, \tag{1}$$

where the subscript 1 indicates quantities measured on the axis.

It is assumed, as usual, that the pressure p is a function of only the longitudinal distance, and can be expressed in terms of λ as follows:

$$p = \mathrm{const}\,\frac{1}{\lambda}\left(1 - \frac{k-1}{k+1}\lambda^2\right).$$

Moreover, using the equation

$$\frac{T_1}{\theta} = 1 - \gamma\lambda_1^2 \qquad \left(\gamma = \frac{k-1}{k+1}\right),$$

where θ is the stagnation temperature, we can readily rewrite Eq. (1) in the form

$$\frac{dS_1}{dx} = \frac{1}{\lambda}\Phi_\omega(\lambda)\frac{d\lambda}{dx} + \frac{2k}{k+1}\frac{\lambda^2}{\omega^3(1-\gamma\lambda_1^2)}\frac{d\omega}{dx}, \tag{2}$$

where $\omega = \lambda/\lambda_1$ is the shape factor of the velocity profile, and

$$\Phi_\omega(\lambda) = \frac{1+\gamma\lambda^2}{1-\gamma\lambda^2} - \frac{2k\lambda^2/(k-1)}{\omega^2 - \gamma\lambda^2}.$$

It is readily verified that, for each fixed ω, the function $\Phi_\omega(\lambda)$ will vanish for some particular value $\lambda = \lambda_\omega$. Moreover, for $\lambda < \lambda_\omega$ the function is positive, and for all $\lambda > \lambda_\omega$ it is negative.

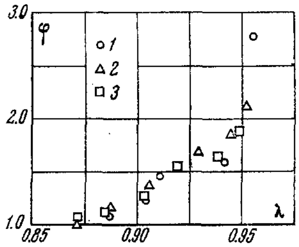

Fig. 2. The quantity φ as a function of the velocity coefficient. The pressure gradients are based on the MO TsKTI data.

Since $dS_1/dx \geq 0$, it follows from Eq. (2) that $d\omega/dx > 0$ for all $\lambda > \lambda_\omega$. Consequently, the velocity profile begins to fill out before λ reaches the value λ_ω. The quantity λ_ω can be found from the equation

$$\Phi_\omega(\lambda) = 0.$$

Hence, we have

$$\lambda_\omega^2 = 4 - {}^1\!/_2\omega^2 - \sqrt{(4 - {}^1\!/_2\,\omega^2)^2 - 6\omega^2},$$

where for air k = 1.4.

For Reynolds numbers R in the range between 10^5 and 10^6 we have $\omega = 0.835$–0.865, which corresponds to $\lambda_\omega = 0.795$–0.830. An appreciable rise in ζ (Fig. 1) was, in fact, observed precisely for λ_ω of this order of magnitude.

We note that the aim of the experiments was not to obtain accurate quantitative results, but to estimate the dependence of ξ on λ for $\lambda \to 1$. However, if, for example, we take into account outflow effects, the value of ξ falls from 2.07 (Fig. 1) to 1.87, i.e., by approximately 10%, but introduces no appreciable modification into the behavior of the coefficient of friction.

We can also estimate τ_0 from data on the static distribution of pressure along the length of the tube. This can be done from an approximate expression for τ which can be used with or without laminarization.

Let us now transform to dimensionless quantities. For the longitudinal distance x, lateral distance y, pressure p, axial velocities u and v, thermodynamic temperature T and stagnation temperature θ, we shall introduce the following characteristic quantities r, rR°, p°, u, u, T, θ, where $R^\circ = (ur/\nu)/\!/2$.

Using the equation of motion for a circular tube in terms of the dimensionless variables

$$\rho\left(u\frac{\partial u}{\partial x} + R^\circ v\frac{\partial u}{\partial y}\right) = -\frac{k-1}{2k}\frac{dp}{dx} + \frac{1}{2r}\frac{\partial(r\tau)}{\partial y},$$

we find that on the tube wall

$$\left(\frac{\partial\tau}{\partial y}\right)_0 = \tau_0 + \frac{k-1}{k}\frac{dp}{dx}. \qquad (3)$$

Let us suppose that on the axis of the tube

$$\left(\frac{\partial\tau}{\partial y}\right)_1 = -f_n\tau_0, \qquad 0 \leqslant f_n \leqslant 1. \qquad (4)$$

Equations (3) and (4), and the symmetry condition ($\tau_1 = 0$), enable us to write τ in the form of the polynomial

$$\tau = \tau_0(1 + a_1y - a_2y^2 + a_3y^3). \qquad (5)$$

From the above three conditions we then have

$$a_1 = 1 + \frac{k-1}{k\tau_0}\frac{dp}{dx},$$

$$a_2 = 5 - f_n + 2\frac{k-1}{k\tau_0}\frac{dp}{dx}, \qquad a^2 = 3 - f_n + \frac{k-1}{k\tau_0}\frac{dp}{dx}. \qquad (6)$$

Integrating Eq. (5) after substitution of the coefficients given by Eq. (6), we obtain

$$\tau_0 > -\frac{k-1}{8k}\frac{dp}{dx} \qquad \left(\int_1^0 \tau\,dy > 0\right), \qquad (7)$$

and therefore

$$\xi = \frac{\tau_0}{w}\frac{w_0}{\tau_{00}} > \varphi = -\frac{k-1}{8k}\frac{w_0}{w\tau_{00}}\frac{dp}{dx},$$

where the subscript 0 indicates the initial cross section of the tube, which was assumed to be at a distance of 50 units from the entrance. Figure 2 shows the calculated values of φ as a function of λ. It is clear that there is a substantial increase in φ and, consequently, in ξ with increasing λ, which is in general agreement with experimental results.

Therefore, the coefficient of friction for sufficiently large $|dp/dx|$ is greater than at the beginning of the developed flow region, regardless of whether or not the laminarization effect occurs.

The authors are grateful to S. S. Kutateladze and A. I. Leont'ev for a number of useful suggestions.

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15 February 1966 Kiev